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ELEVENTH PROOF. — The circle which circumscribes the triangle ABC is the nine point circle of the triangle $O'O''O'''$ and consequently bisects $O''O'''$ and also OO'. (O denoting the intersection of the lines AO', BO'', CO'''.) Because M is the middle point of $O''O'''$

$$MP = \tfrac{1}{2}(r''+r'''),$$

and because N is the middle point of OO'

$$NP = \tfrac{1}{2}(r' - r).$$

Adding $$MP+NP = 2R = \tfrac{1}{2}(r'+r''+r'''-r),$$

$$\therefore\ 4R = r'+r''+r'''-r.$$

A PROBLEM WITH SOLUTION.

BY A. S. HATHAWAY, CORNELL UNIV., ITHACA, N. Y.

LET u', u'', . . . , v', v'', . . . be such functions of x that each vanishes when $x = a$; and also the derivatives of each, up to (but not including) those of s', s'', . . . , t', t'', . . . order, respectively. It is then required to find the limit as x approaches a, of $\dfrac{u'u''\ldots}{v'v''\ldots}$.

We proceed to find the order and value of the first non-vanishing derivative of each product.

Since differentiation always produces expressions which are homogeneous with respect to the introduced $\dfrac{d}{dx}$, the $s'+s''+\ldots.$ derivative of $u'u''\ldots.$ contains the term $A\left(\dfrac{d}{da}\right)^{s'}u'\left(\dfrac{d}{dx}\right)^{s''}u''\ldots.$ which does not vanish; and besides, other terms of equal order, cannot be without a vanishing derivative, and are all zero, unless some other derivative of not greater than order $s'+s''+\ldots.$ is infinite. This indeterminate case we will exclude; and then all terms of less order also vanish.

We find $A = \dfrac{(s'+s''+\ldots)!}{s'!\,s''!\ldots}$, for in repeated differentiations of a product, the combinations of partial derivatives are the same as in the case of powers of a multinomial algebraic expression; as is evident on considering that

$$\frac{d}{dx}u'u''\ldots = \left(\frac{d}{dx'}+\frac{d}{dx''}+\ldots\right)u'u''. \quad \text{(See NOTE.)}$$

Similar results hold for $v'v''$ . . . . , with the restriction that no derivative of v' or v'' . . . , of not greater order than $t'+t''+\ldots$ is infinite; and the coefficient of the term which does not vanish $= \dfrac{(t'+t''+\ldots)!}{t'!\,t''!\ldots}$.

Therefore, using D for $\frac{d}{dx}$, and *a' b* for *a or b*,

$$\lim\frac{u'u''\ldots}{v'v'\ldots} = 0'\ \lim\left(\frac{D^{s'}u'\,D^{s''}u''\ldots}{s'!\,s''!\ldots} \div \frac{D^{t'}v'\,D^{t''}v''\ldots}{t'!\,t''!\ldots}\right)'\ \infty,$$

according as $s'+s''+\ldots >, =$ or $< t'+t''+\ldots$ (1)

A special case of this problem is to find, dropping the accents, limit $\frac{u^m}{v^n}$, where m and n are any quantities.

We have the identity $\lim\frac{u^m}{v^n} = \lim\left(\frac{u^t}{v^s}\right)^{\frac{n}{s}} \lim u^{\frac{ms-nt}{s}}$, and this is, if no derivative of u or v of not greater order than st is infinite,

$$0'\ \lim\left[\left(\frac{D^s u}{s!}\right)^m \div \left(\frac{D^t v}{t!}\right)^n\right]'\ \infty, \text{ according as } ms >, = \text{ or } < nt. \qquad (2)$$

Note —We may find A inductively, using Leibnitz's Theeorem. Thus, when $a = a$,

$$D^{s'}u' = \frac{s'!}{s'!}D^{s'}u'$$

$$D^{s'+s''}u'u'' = \frac{(s'+s'')\ldots(s'+1)}{s''!}D^{s'}u'D^{s''}u'' = \frac{(s'+s'')!}{s'!\,s''!}D^{s'}u'D^{s''}u''$$

$$D^{s'+s''+s'''}u'u''u''' = \frac{(s'+s''+s''')\ldots(s'+s''+1)}{s'''!}D^{s'+s''}u'u''D^{s'''}u'''$$

$$= \frac{(s'+s''+s''')!}{s'!\,s''!\,s'''!}D^{s'}u'D^{s''}u''D^{s'''}u'''$$

. . . .

Examples.

$$\lim\frac{\sin^3(a-1)[2\cos\tfrac{1}{2}\pi a+\pi(a-1)][2\cos(a-1)-\cos^2(a-1)+\tfrac{1}{4}(a-1)^4]}{[(\log a)^2-(a-1)^2\div a][2\cos(1-a)+a^2-2a-1][8\varepsilon^{a-1}-\varepsilon^{2a-2}-2a(a+1)-3]}$$

$$= (a=1)\ \tfrac{3}{4}\pi^3.$$

$$\frac{[a-1-\sin(a-1)]^{\frac{1}{2}}}{[1-\cos(a-1)]^{\frac{3}{4}}} = (a=1)\ [\tfrac{1}{3}\sqrt{2}]^{\frac{1}{2}};$$ Ex. 55, chap. 10, Tod. Dif. Cal.

DEMONSTRATION OF THE PROP. AT PAGE 8.

BY PROF. D. J. MC. ADAM, WASHINGTON, PA.

Let *B CD A* be a semicircle, diameter *AB*, *ABCD* being any inscribed trapezium, to prove that the sides *AD*, *DC* and *CB* may represent the reciprocals of the lines *a*, *b*, *c* if *AB* represent the reciprocal of *r*.